STORED ACTIVITIES

spike number
\[ z \in \{0,1\} \]

spike phase \( x \)
STORED ACTIVITIES

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\[ z \in \{0,1\} \]

spike phase \( x \)

\[ w_{ij} = \sum_{m=1}^{M} z_i z_j \Omega \left( x_i^{(m)}, x_j^{(m)} \right) \]
REPRESENTING UNCERTAINTY: FIRING RATES AND PHASES

Lengyel & Dayan, NIPS 2006

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STORED ACTIVITIES

- Spike number: \( z \in \{0,1\} \)
- Spike phase: \( x \)

RECALLED ACTIVITIES

- Spike number: \( n \geq 0 \)
- Mean phase: \( \varphi \)
- Concentration: \( \epsilon \)
REPRESENTING UNCERTAINTY: FIRING RATES AND PHASES

Lengyel & Dayan, NIPS 2006

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RECALLED ACTIVITIES

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\( \varphi \)

concentration
\( c \)

\[
\begin{align*}
\omega_{ij} = \sum_{m=1}^{M} z_i z_j \Omega(x^{(m)}_i, x^{(m)}_j)
\end{align*}
\]
true solution: \[ P(z, x | \tilde{x}, W) \]
true solution: \( P(z, x|\tilde{x}, W) \)

variational approximation: \( Q(z, x; n, \phi, c) = \prod_i q(z_i, x_i; n_i, \phi_i, c_i) \)
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adjust spike numbers, phases and concentrations so that \( Q \) best matches \( P \)
true solution: \( P(z, x|\tilde{x}, W) \)

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adjust spike numbers, phases and concentrations so that $Q$ best matches $P$

network dynamics implements gradient ascent on KL divergence between $Q$ and $P$

$$
\frac{d}{dt} n_i \propto \frac{\partial}{\partial n_i} \text{KL}[Q(z, x; n, \phi, c) || P(z, x|\tilde{x}, W)]
$$

$$
\frac{d}{dt} \phi_i \propto \frac{\partial}{\partial \phi_i} \text{KL}[Q(z, x; n, \phi, c) || P(z, x|\tilde{x}, W)]
$$

$$
\frac{d}{dt} c_i \propto \frac{\partial}{\partial c_i} \text{KL}[Q(z, x; n, \phi, c) || P(z, x|\tilde{x}, W)]
$$
true solution: \( P(z, x|\tilde{x}, W) \)

variational approximation: \( Q(z, x; n, \phi, c) = \prod_i q(z_i, x_i; n_i, \phi_i, c_i) \)

**mean phase dynamics**

\[
\frac{d}{dt} \phi_i \propto \ldots + \sum_j n_j c_j w_{ij} \frac{\partial}{\partial \phi_i} \Omega(\phi_i, \phi_j)
\]

same as before + modulation by burst strength

adjust **spike numbers**, **phases** and **concentrations** so that \( Q \) best matches \( P \)
a single trial
PERFORMANCE OF THE RATE & PHASE NETWORK
Lengyel & Dayan, NIPS 2006

a single trial

recall of phases

Máté Lengyel: Episodic memory: why and how?
PERFORMANCE OF THE RATE & PHASE NETWORK
Lengyel & Dayan, NIPS 2006

a single trial

recall of phases

recall of rates
TESTING THE PREDICTIONS
PRELIMINARY IN VITRO DATA

Máté Lengyel: Episodic memory: why and how?

http://www.eng.cam.ac.uk/~m.lengyel
UNCERTAINTY SIGNALS PREDICT ERROR
Lengyel & Dayan, NIPS 2006
UNCERTAINTY SIGNALS PREDICT ERROR
Lengyel & Dayan, NIPS 2006

burst strength → phase errors

burst strength
- 0.05
- 0.2
- 0.4
- 0.7

Frequency

Error in firing phase

0 0
-π π
UNCERTAINTY SIGNALS PREDICT ERROR
Lengyel & Dayan, NIPS 2006

burst strength $\rightarrow$ phase errors
firing rate $\rightarrow$ rate errors

burst strength
- 0.05
- 0.2
- 0.4
- 0.7

Error in firing phase

Frequency

0.6
0.5
0.4
0.3
0.2
0.1
0

-\pi
0
\pi

Stored firing rate

0
0.2
0.4
0.6
0.8
1

0
0.2
0.4
0.6
0.8
1

Retrieved firing rate
UNCERTAINTY SIGNALS PREDICT ERROR
Lengyel & Dayan, NIPS 2006

burst strength $\rightarrow$ phase errors

firing rate $\rightarrow$ rate errors

in vitro data

Lengyel & Dayan, NIPS 2006

in vitro data
CONCLUSIONS
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- normative models can go a long way to understand the fundamental challenges involved in learning and memory ... ... and how the brain might solve them
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  - is appropriate for formalising the computational task
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- learning and memory are not the same

- even episodic memory retrieval is a probabilistic inference task
Máté Lengyel: Episodic memory: why and how?

BCCN 2009, 3 October 2009

http://www.eng.cam.ac.uk/~m.lengyel

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**theory**  

**in vitro experiments**

**in vivo experiments**

postdoc position available, see
[http://learning.eng.cam.ac.uk](http://learning.eng.cam.ac.uk)
OPTIMAL PHASE-BASED INTERACTION: THE PHASE RESPONSE CURVE

spike timing-dependent plasticity: $\Omega$

coupling function: $H$

phase response curves

- $w=1$
- $w=2$
- $w=3$
- $w=4$
ROBUSTNESS OF RECALL PERFORMANCE

asymmetric STDP

Synaptic weight change

Average error (rad)

$t_{\text{pre}} - t_{\text{post}}$ (ms)

Number of stored memories

- input only
- antisym, matched
- weakly asym, non-matched
- strongly asym, matched
ROBUSTNESS OF RECALL PERFORMANCE

asymmetric STDP

![Graph showing synaptic weight change over time](image)

![Graph showing average error over number of stored memories](image)

sparse connectivity

![Graph showing average error over number of stored memories](image)
ROBUSTNESS OF RECALL PERFORMANCE

asymmetric STDP

sparse connectivity

storage noise

Máté Lengyel: Optimal memory storage in neural networks
http://www.eng.cam.ac.uk/~m.lengyel

Budapest Computational Neuroscience Forum, 5 January 2009
BURST STRENGTH \((\text{SPIKE NUM} \times \text{CONC}) = \text{CONFIDENCE}\)  

IN VIVO DATA
BURST STRENGTH (SPIKE NUM × CONC) = CONFIDENCE

IN VIVO DATA
BURST STRENGTH (SPIKE NUM × CONC) = CONFIDENCE

IN VIVO DATA

- Black circles: spikes
- Red circles: mean phase within theta cycle
BURST STRENGTH (SPIKE NUM × CONC) = CONFIDENCE

*IN VIVO DATA*
BURST STRENGTH (SPIKE NUM $\times$ CONC) = CONFIDENCE

IN VIVO DATA

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IN VIVO DATA

- spikes
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IN VIVO DATA

Position

Phase

● spikes

mean phase within theta cycle

‘true’ phase = avg. of mean phases across trials
BURST STRENGTH (SPIKE NUM × CONC) = CONFIDENCE

IN VIVO DATA

- spikes
- mean phase within theta cycle
- ‘true’ phase = avg. of mean phases across trials
- ‘error’ = mean - ‘true’ phase
BURST STRENGTH (SPIKE NUM × CONC) = CONFIDENCE
*IN VIVO DATA*

- **Phase**
- **Position**

- **Spikes**
- **Mean phase within theta cycle**
- **'True' phase** = avg. of mean phases across trials
- **'Error'** = mean - 'true' phase

---

**Burst strength** (spikes / cycle):
- 0–0.5
- 0.5–1.5
- 1.5–2.5
- 2.5–3.5
- 3.5–4.5

**Frequency**

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http://www.eng.cam.ac.uk/~m.lengyel