

1 **Supplementary material**

2 **The statistics of natural movements are reflected in motor errors**

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9 *Wavelet phase analysis*

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11 To determine the phase relations between movements of the left and right arms as a function
12 of frequency, we performed a Dual-Tree Complex Wavelet Transform (DT-CWT), as
13 described in the Methods of the main text. We generated phase incidence histograms by
14 binning the relative phases into 18 bins each 20° wide. The total incidences across all bins in
15 a given frequency band were then normalized to sum to 1. We validated this analysis on two
16 different datasets. For the first dataset, we generated synthetic left and right elbow angle data
17 by simulating two independent random walks. For the second dataset, we inserted a set of
18 relative time shifts between the left and right elbow angles from the natural movement
19 dataset. In all cases the data was analyzed at low and high frequency bands, as described in
20 the main text. That is, 0.45-0.9 Hz for the low frequency band and 3.2-6.4 Hz for the high
21 frequency band.

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23 *Validation using a random walk dataset*

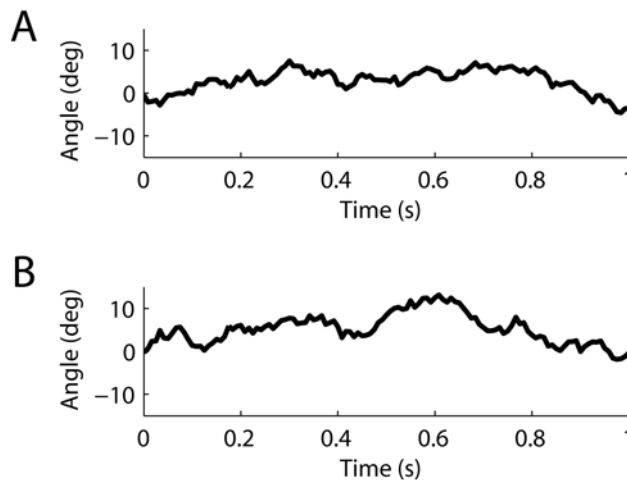
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25 The first validation dataset simulated the angular trajectories of the left and right elbows using
26 two independent random walks. Approximately 18 hours of synthetic data was generated by
27 cumulatively summing 8×10^6 samples (assuming a 120 Hz sampling rate) which were drawn
28 from a zero mean Gaussian distribution with unit variance. The initial 1 second of data is
29 shown in Figure S1.

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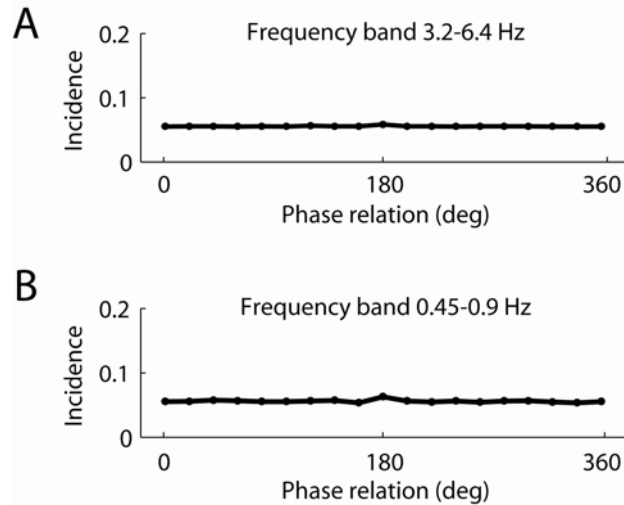
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Figure S1. Synthetic random walk elbow angle data for the left (A) and right (B) elbows.

37 The wavelet relative phase analysis was run on this dataset and the resulting relative
38 incidence of phase occurrences are shown in Figure S2. It can be seen that the relative
39 incidence of phase is essentially uniformly distributed at both high (Figure S2 A) and low
40 frequencies (Figure S2 B), as expected from two independent random signals.
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Figure S2 Relative phase analysis for the synthetic random walk dataset. **A** Normalized phase incidences at 3.2-6.4 Hz (high frequency band). **B** Normalized phase incidences at 0.45-0.9 Hz (low frequency band).

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50 *Validation using time shifted natural movement dataset*

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52 The second validation dataset was generated by introducing a set of time shifts between the
53 left and right elbow angle time series from the natural movement dataset. The effect of such a
54 time shift is to change the phase relations between the left and right datasets in a frequency
55 dependent fashion. We note that only time shifts that lead to small phase shifts (for example,
56 less than 180°) lead to meaningful interpretations. This is because for larger phase shifts, we
57 should expect the phase relationships to tend towards a uniform distribution since the
58 movements of the left and right elbow will no longer correspond to the same action in time.

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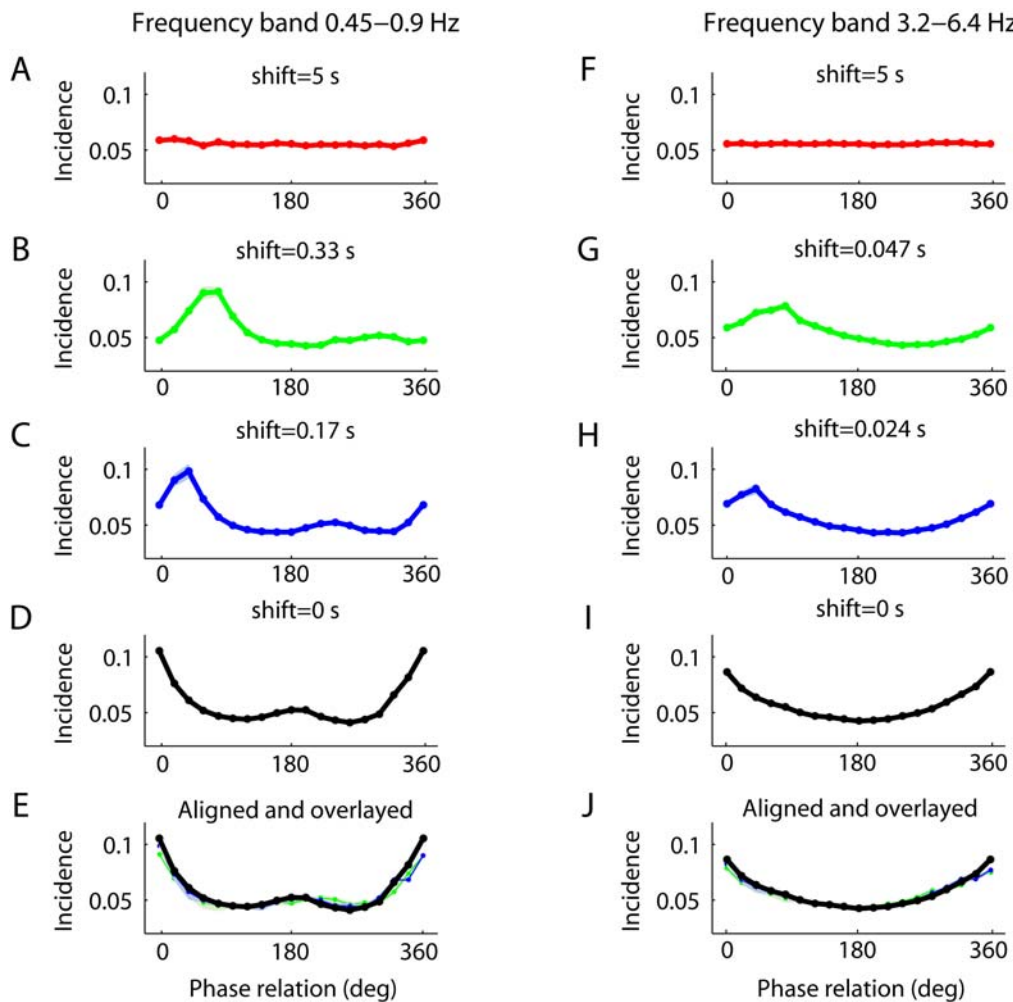
60 We selected 6 time shifts which include the two extremes cases of zero (the normal non-
61 shifted case used in the main analysis) and a large time shift of 5 s which, as discussed above,
62 should remove the correspondence between movements of the right and left elbow and result
63 in a uniform distribution of phase relations. The additional time shifts were selected to give
64 rise to specific phase shifts for the high and low frequency bands. Since our phase bins
65 increase in steps of 20°, we then select time delays that would lead to phase shifts that
66 correspond to this bin width.

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68 Two time shifts were selected to correspond to 40° and 80° phase shifts at the low frequency
69 band, as follows. The frequency at the centre of this band is 0.67 Hz, which corresponds to a
70 period of 1.5 s. Therefore, a phase shift of 40° corresponds to a time shift of 0.17 s and a
71 phase shift of 80° corresponds to a time shift of 0.33 s. The two time shifts for the high
72 frequency band were similarly selected to correspond to 40° and 80° phase shifts. The
73 frequency at the centre of this band is 4.78 Hz, which corresponds to a period of 0.21 s.
74 Therefore, a phase shift of 40° corresponds to a time shift of 0.023 s and a phase shift of 80°
75 corresponds to a time shift of 0.047 s.

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77 The wavelet relative phase analysis was run on the time-shifted datasets for the 6 time-shift
 78 values and across the 6 subjects. The results are shown in Figure S3. At the large time shift of
 79 5 s, the phase relation at both low and high frequencies was uniform (Figure S3 A and F).
 80 This is to be expected because at such a large time shift, movements of left and right elbows
 81 will be unrelated. For time shifts corresponding to an 80° phase shift, it can be seen that the
 82 peak in incidence that occurs at 0° in the non-shifted data, has been shifted towards 80°, for
 83 both the low (Figure S3 B) and high frequency bands (Figure S3 G). A similar effect occurs
 84 for time shifts corresponding to a 40° phase shift (Figure S3 C and H). For comparison the
 85 zero time shift incidences are shown (Figure S3 D and I). Finally, results for the 40° and 80°
 86 phase shifts for the low and high frequency bands were aligned by shifting each curve by its
 87 expected phase shift and overlaying on the original non-shifted analysis (Figures S3 E and J).
 88 It can be seen that there is a good agreement between these curves, indicating that the relative
 89 time shifts have resulted in the expected phase shifts.



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Figure S3 Validation of wavelet phase analysis with a dataset generated by inserting a relative time shift between the left and right elbow angle data from the natural movement dataset, averaged over all six subjects. **A-E** Relative phase incident at 0.45-0.9 Hz (low frequency band). **F-J** Relative phase incident at 3.2-6.4 Hz (high frequency band) **A & E** For a large time shift (5 s), relative phase incidence is uniform. **B & G** A time shift corresponding to 80° results in the expected phase shift (see text for details). **C & H** A time shift corresponding to 40° results in the expected phase shift (see text for details). **D & I** The relative phase incidences for zero time shift. **E & J** Good agreement of curves is

101 obtained when the relative phase incidences for different time shifts are aligned and
102 overlaid with the zero time shift data.

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104 *Conclusions*

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106 To summarize, analysis of a random walk dataset yielded a uniform phase incidence and
107 inserting various relative time shifts in the natural movement dataset resulted in the expected
108 shifts relative phase incidence. These two experiments thus demonstrate the validity of the
109 wavelet phase analysis that was used in the analysis of the natural movement dataset
110 described in the main text.

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